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AN EXTENSION OF EXISTING SOLIDIFICATION RESULTS OBTAINED FROM THE HEAT-BALANCE INTEGRAL METHOD

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POOTS [1, 2] has applied the heat-balance method to the problems of freezing of circular cylinders, spheres and uniform prisms. However by applying simple transformations, it is possible to extend his results to the more general shapes of ellipses, rectangular prisms and ellipsoids.

ELLIPSE AND ELLIPSOID

For the circular cylinder and sphere problems Poots assumed that the distribution for the normalised temperature was given by (nomenclature as in previous paper [3])

$$\theta = \frac{X}{\varepsilon} \quad \text{(one-parameter method)}$$

or

$$\theta = \frac{X}{\varepsilon} + g\left(\frac{X}{\varepsilon} - \frac{X^2}{\varepsilon^2}\right) \quad \text{(two-parameter method)}$$

where $X = 1 - r_\lambda/a$ (r_λ: appropriate radial coordinate with $\lambda = 1$ and 2 for the cylinder and sphere, respectively and a is the radius) and g and ε are parameters determined by heat-balance integral equations; the solidification front is assumed to be given by $R = \varepsilon$. Now if instead we want to consider an elliptical cylinder or ellipsoid, then all we have to do is make a transformation from Poots τ to τ^* via

$$\tau^* = \tfrac{1}{2}\tau\left[1 + \left(\frac{a}{b}\right)^2\right] \tag{1}$$

in the case of the elliptical cylinder, or

$$\tau^* = \tfrac{1}{3}\tau\left[1 + \left(\frac{a}{b}\right)^2 + \left(\frac{a}{c}\right)^2\right]$$

in the case of the ellipsoid, and Poots' results for the circular cylinder and sphere can be used. The results correspond to using the same form of profile but with

$$X = 1 - \sqrt{\left[\frac{x^2}{a^2} + \frac{y^2}{b^2}\right]}$$

and

$$X = 1 - \sqrt{\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right]}$$

in the respective cases. Here x, y, z are the usual cartesian coordinates, a is the length of the major axis and b and c the lengths of the minor axes.

UNIFORM RECTANGULAR PRISM

Poots treated the problem of the solidification of the uniform square prism of side $2a$ by assuming temperature profiles of the form

$$\theta = \frac{R}{\varepsilon} \quad \text{(one-parameter method)}$$

$$= (1-g)\frac{R}{\varepsilon} + g\left(\frac{R}{\varepsilon}\right)^2 \quad \text{(two-parameter method)}$$

where $R = (1-X^2)(1-Y^2)$ and $X = x/a$, $Y = y/a$, and $R = \varepsilon$ defines the solidification front. If we assume a similar temperature profile except with

$$X = x/a \qquad Y = y/b$$

then Poots' results for the uniform prism may be simply extended to a rectangular prism of sides $2a$, $2b$ by replacing τ by τ^* given by (1).

The above procedure has also been used in a companion paper [3] to generalise the results for cubes to cuboids.

COMMENT

It should be stressed that it is always possible to generalise, e.g. from spheres to ellipsoids or cubes to cuboids, etc., the results concerning the solidification front provided that the assumed temperature profile and interface, described in non-dimensional variables are the same in the two cases. This is true whatever the assumed profile.

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